Accelerated MR Diffusion Tensor Imaging Using Distributed Compressed Sensing

Yin Wu,1,2 Yan-Jie Zhu,1,2 Qiu-Yang Tang,1,2 Chao Zou,1,2 Wei Liu,1,2 Rui-Bin Dai,1,2 Xin Liu,1,2 Ed X. Wu,3,4 Leslie Ying,5,6 and Dong Liang1,2*

Purpose: Diffusion tensor imaging (DTI) is known to suffer from long acquisition time in the orders of several minutes or even hours. Therefore, a feasible way to accelerate DTI data acquisition is highly desirable. In this article, the feasibility and efficacy of distributed compressed sensing to fast DTI is investigated by exploiting the joint sparsity prior in diffusion-weighted images.

Methods: Fully sampled DTI datasets were obtained from both simulated phantom and experimental heart sample, with diffusion gradient applied in six directions. The k-space data were undersampled retrospectively with acceleration factors from 2 to 6. Diffusion-weighted images were reconstructed by solving an l2/l1 norm minimization problem. Reconstruction performance with varied signal-to-noise ratio and acceleration factors were evaluated by root-mean-square error and maps of reconstructed DTI indices.

Results: Superiority of distributed compressed sensing over basic compressed sensing was confirmed with simulation, and the reconstruction accuracy was influenced by signal-to-noise ratio and acceleration factors. Experimental results demonstrate that DTI indices including fractional anisotropy, mean diffusivities, and orientation of primary eigenvector can be obtained with high accuracy at acceleration factors up to 4.

Conclusion: Distributed compressed sensing is shown to be able to accelerate DTI and may be used to reduce DTI acquisition time practically.

Key words: diffusion tensor imaging; distributed compressed sensing; joint sparsity constraint; fast imaging

Magnetic resonance (MR) diffusion tensor imaging (DTI) has been widely used as a powerful tool to nondestructively probe biological tissue structures such as myocardium with high spatial resolution. Direct correlation between DTI and histological fiber direction measurements in myocardium has been demonstrated, validating the DTI approach to delineate tissue fiber architecture (1–3). So far, DTI has been regarded as the only means to observe diffusion in vivo noninvasively, without interfering with the diffusion process itself (4).

A minimum of seven scans (six diffusion-weighted images (DWIs) and one image without diffusion weighting) is required to fully determine the diffusion tensor. This minimal set of acquisitions may be repeated for averaging to achieve sufficient signal-to-noise ratio (SNR) (5,6). Therefore, most DTI study suffers from lengthy data acquisition, which greatly limits its practical and clinical utilities. For instance, imaging on ex vivo heart may take several hours using a 2D or 3D spin-echo DTI sequence (7–12) to achieve subtle spatial resolution (at orders of μm²), which is advantageous to investigations of myocardial microstructure. Even by incorporating echo-planar imaging and parallel imaging technique, the acquisition time can still be around 1 h to achieve an isotropic resolution of 1.13 mm³ (13–16). Recently, successful implementations of DTI on in vivo human heart were reported (17–20). However, approximately a time of 10 min was generally needed for acquiring one slice with a spatial resolution of 2 × 2 × 8 mm³ when six diffusion directions were used (19,20). The limited temporal and spatial resolution substantially degraded the efficiency in clinical diagnosis and evaluation. Thus, a feasible way to accelerate DTI data acquisition while maintaining high resolution is highly desirable in order to broaden its imaging capability and practical applications.

Hsu and Henriquez (21) successfully doubled the acquisition rate by applying the principles of reduced encoding imaging with different types of reconstruction schemes. Subsequently, a filtered reduced-encoding projection-reconstruction technique was introduced to further improve the image reconstructions (22). Aiming to
resolve the limited SNR problem, Haldar et al. developed a joint feature-preserving regularized image reconstruction method in various diffusion imaging applications, particularly when a high b-value was used (23–26). Assuming signal smoothness in most regions and highly correlation of edge structures in DWIs, this specialized denoising scheme has been demonstrated to efficiently achieve robust and accurate DTI index estimation from a small number of DWIs (27) or scan repetitions (24), reducing data acquisition time. It is worth noting that this method should more appropriately be classified as regularized reconstruction rather than reduced-encoding framework because the problem was defined in the fully sampled k-space.

In recent years, compressed sensing (CS) has emerged as a new framework for fast data acquisition (28,29). The basic CS theory allows images to be recovered from randomly undersampled k-space data using a nonlinear reconstruction algorithm, which only enforces data consistency and transform-sparsity in MR images. Based on this principle, several methods were derived to accelerate DTI acquisition without compromising the reconstruction accuracy, such as adding constraints along both spatial and diffusion directional dimensions (30) or on tensor and phase (31). The greatest acceleration factors achieved were 2 or 4 using Cartesian or radial sampling trajectories, respectively. Most recently, a method of mean-based CS has been applied to fast DTI acquisition (32), where it was observed that isotropic components presented similar signal intensities in all DWIs. The images could be jointly recovered by updating the innovation part with diffusivity after linearly reconstructing the common component. Wavelet sparsity of coefficients of principal component analysis along diffusion direction domain could also be used to reduce imaging time (33). Superior reconstruction fidelity was generally attained to that of basic CS method.

More complex diffusion methods or models, such as diffusion spectrum imaging and high angular resolution diffusion imaging for addressing the “crossing-fiber” problem, also suffer from lengthy acquisition time. With the confirmation of the sparse characteristics of diffusion propagators (33,34) or numbers of fiber orientations (35–37), acceleration of data acquisition has been realized by using CS in an analysis or synthesis way (38,39).

With additional priors besides sparsity, some CS extensions exhibit better performance than those of basic CS in various MR applications (40,41). One such approach is known as distributed CS (DCS). This approach exploits intra- and inter-signal correlation structures simultaneously within one constraint term when signals are sampled from multiple channels (42). DWIs obtained from different diffusion directions can be regarded as signals from multiple channels, making DCS a promising approach for DTI acceleration. This was not fully used in previous fast DTI studies. In this study, we proposed a novel formulation (i.e., DCS-based reconstruction) to simultaneously recover diffusion signals with fully utilizing the joint sparsity property (i.e., sharing the common support) among DWIs. Its feasibility and efficacy in accelerating conventional DTI acquisition were extensively explored using both simulation and MR experiments for the first time. A part of the preliminary work has been presented in ISMRM 2012 Annual Meeting and Exhibition (43). Although the approach was studied with conventional DTI, it may be readily adopted by the aforementioned complex diffusion models and methods.

METHODS

Accelerating DTI Using Distributed Compressed Sensing

DCS, as an extension of CS by incorporating the joint sparsity prior of multiple sparse signals, has been exploited in numerous studies (42,44–46). It is shown through theoretical analysis and numerical study that this new technique can effectively reduce the number of measurements to achieve a given reconstruction quality or improve the reconstruction quality for a given number of measurements, compared with the basic disjoint CS method of reconstructing signals individually without using their joint sparsity property.

It is known that sampling multiple DWIs can be formulated as

$$y_l = F_l x_l, \quad l = 1, 2, \cdots, L$$

where $L$ is the number of diffusion directions, $F_l$ is the undersampled Fourier matrix for the $l$th direction with size $m \times n$, and $y_l$ is the k-space measurements of the $l$th direction. If the above samplings satisfy CS conditions given in (29), the image $x_l$ can be reconstructed separately using the basic CS method by solving the following optimization problem

$$\min \|\Psi x_l\|_1 \quad s.t. \quad \|y_l - F_l x_l\|_2 \leq \varepsilon \tag{2}$$

where $\Psi$ denotes the sparsifying transform and $\varepsilon$ is the noise level.

Considering that directional diffusion gradients modulate the magnitudes of DWIs, joint sparsity property should exist among the images. This means that each image $x_l$ is transform-sparse and shares the same sparse support in the sparsifying transform domain with other images. Therefore, DCS is an appropriate tool to deal with this kind of reconstruction case. Equation [1] could be rewritten as $y = Fx$, i.e.,

$$\begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_L
\end{bmatrix} =
\begin{bmatrix}
F_1 & 0 & 0 & 0 \\
0 & F_2 & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & F_L
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_L
\end{bmatrix} \tag{3}$$

Then, all $x_l$ can be reconstructed simultaneously by solving a row-$\ell_0$ minimization problem (47):

$$\min (\text{# of nonzero rows in } x) \quad s.t. \quad y = Fx \tag{4}$$

where $x = [x_1, x_2, \cdots, x_L]$ is a 2D matrix with size of $n \times L$. Greedy algorithm or convex relaxation can be used to solve this reconstruction problem (47,48), because
solving Eq. [4] is NP-hard. In this study, the convex relaxation method was used to reconstruct \( x \) by minimizing an \( l_2-l_1 \) norm

\[
\min \sum_{i=1}^{n} ||B_i||_2 \quad s.t. \quad ||y-F^*x||_2 \leq \epsilon \tag{5}
\]

where \( B_i \) is the \( i \)-th row of coefficients matrix \( B = \Psi \tilde{x} \). This objective function is equivalent to initially applying the \( l_2 \) norm to rows (i.e., to promote nonsparsity along diffusion directions) and then applying the \( l_1 \) norm to the resulting vector (i.e., to promote sparsity along spatial direction). It suggests we want most rows of the coefficients matrix should be zero, but the nonzero rows should have many nonzero entries.

The constrained minimization in Eq. [5] is usually obtained by solving an equivalent unconstrained regularization problem:

\[
\arg\min \left( ||y-F^*x||_2^2 + \lambda \sum_{i=1}^{n} ||B_i||_2 \right) \tag{6}
\]

where \( \lambda \) is the regularization parameter for balancing the regularization term and the data consistency term.

**Numerical Simulation**

A computer simulation was implemented to mimic a DTI experiment. Six diffusion gradients at directions of \([0 \pm 0.618 \pm 1], [0.618 \pm 0], \) and \([-1 0 \pm 0.618]\) with diffusion sensitivity \( b \)-value = 0 and 1000 s/mm\(^2\) were applied on a two-compartment simulated phantom. The primary eigenvalues were set to be \( 1.0 \times 10^{-2} \) and \( 0.8 \times 10^{-3} \) mm\(^2\)/s, secondary eigenvalues were \( 0.6 \times 10^{-3} \) and \( 0.5 \times 10^{-3} \) mm\(^2\)/s, and tertiary eigenvalues were \( 0.4 \times 10^{-3} \) and \( 0.2 \times 10^{-3} \) mm\(^2\)/s for the outer and inner compartment, respectively. The matrix size was set to be 256 \( \times \) 256. The complex gaussian-white noise was added to simulate noisy data with SNR of 40 dB, 30 dB, and 20 dB on averaged DWIs, respectively.

**Heart Sample DTI Experiment**

The animal experiment was approved by the local institutional ethics committee. The imaging experiment was conducted on a 7 T Bruker PharmaScan (Bruker BioSpin) under room temperature \( \sim 20 \) C. A formalin-fixed rat heart sample was contained in a tube and imaged with multislice spin-echo DTI along the short-axis orientation. Imaging parameters were: pulse repetition time/echo time = 1500/29 ms, field of view = 2.55 \( \times \) 2.55 cm\(^2\), slice number = 3, \( b \)-value = 0 and 1000 s/mm\(^2\), matrix size = 256 \( \times \) 256, diffusion gradient duration = 3 ms, diffusion gradient number = 6, and number of repetitions = 10. Slice thickness was 1.5 mm with slice gap of 0.3 mm. Total image acquisition time was \( \sim 7 \) h.

**Data Processing**

Cartesian undersampled complex DTI datasets were obtained retrospectively from the fully sampled data, which served as the gold standard for evaluation of subsequent reconstruction performance. The probability density function (PDF), controlling the sampling pattern of k-space along phase-encoding direction, was constructed by following a polynomial variable density function as indicated below (29):

\[
PDF = (1-r)^p \tag{7}
\]

where \( r \) is the fully sampled percentage in k-space center and \( p \) is the polynomial order. In this study, \( r \) was set to be 20% for acceleration factors of 2 and 3, 15% for 4, and 10% for 5 and 6, respectively. \( p \) was empirically determined to be 1 larger than the respective acceleration factor (for example, \( p = 5 \) for an acceleration factor of 4).

It should be noted that the k-space of the image with \( b \)-value of 0 (i.e., \( b_0 \) image) was fully sampled in this study. In addition, the individual sampling mask following the same PDF was generated for each DWI corresponding to a given acceleration factor \( R \), which helped to further ensure the incoherence.

Based on our preliminary trials, the spatial sparsifying transform \( \psi \) was empirically chosen to be Daubechies-4 wavelet transform with three-level decomposition. However, other nonorthogonal transformations can also be used. Nonlinear conjugate gradient algorithm was used to solve Eq. [6] (29). In general, three iterations were found to be enough to achieve a good reconstruction performance. The regularization parameter was empirically chosen to minimize root-mean-square error (RMSE) of the reconstructed DTI indices as follows:

\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} ||x_{acc}(i) - x_{full}(i)||^2} \tag{8}
\]

where \( x_{acc} \) is the accelerated reconstruction, \( x_{full} \) corresponds to the Fourier reconstruction from fully sampled data, and \( N \) is the total number of pixels inside of the region of interest. Smoothing was applied when calculating the derivative of the cost function as described in (29).

For the purpose of comparison, basic CS method was also performed to reconstruct DWIs one by one from the same undersampled data, without using the joint sparsity property. Additionally, equivalent acceleration could also be achieved by simply decreasing the scan repetition number. In our study, the derived DTI indices were also calculated from such repetition-reduced method with denoising (25) for comparison. The calculated DTI indices using both the DCS-based and repetition-reduced methods were compared at acceleration factors of 2 and 5. Specifically, given that orthogonal wavelets (DB4) were used, Eq. [6] was reformulated as a denoising problem by simply setting \( F \) to the identity operator. This is then solved noniteratively using joint soft-thresholding (49). The regularization parameter was manually selected to minimize RMSE. All reconstruction methods were implemented in MATLAB on a workstation with 3.47 GHz GPU and 64 GB RAM.

For the original and undersampled/repetition-reduced data, the three major eigenvalues (denoted as \( \lambda_1, \lambda_2, \) and \( \lambda_3 \)) together with the primary eigenvector representing
fiber orientation were subsequently calculated from the reconstructions. Fractional anisotropy (FA) and mean diffusivity (MD), resembling fiber integrity and average diffusion rate (50), were calculated as:

$$MD = \frac{\lambda_1 + \lambda_2 + \lambda_3}{3}$$

$$FA = \sqrt{3 \left[ (\lambda_1 - MD)^2 + (\lambda_2 - MD)^2 + (\lambda_3 - MD)^2 \right] / 2 (\lambda_1^2 + \lambda_2^2 + \lambda_3^2)}$$

The helix angle, defined as the angle between the projections of the primary eigenvector onto the tangent plane and the image plane, was also computed from the primary eigenvector as described by Scollan et al. (1) and Chen et al. (9). Such double-helical arrangement of left ventricular (LV) myocardial fibers has been demonstrated to be efficient for dispersing strain uniformly and conserving energy expenditure (51). RMSEs of FA, MD, and angular deviation of myocardial fiber orientation were computed.

**RESULTS**

Sampling masks for DWIs along phase-encoding and diffusion directions at different acceleration factors are displayed in Figure 1. Diffusion data along the frequency-encoding direction, which was perpendicular to the above mentioned two directions, were fully sampled. The masks were different along diffusion directions to further ensure the incoherence requirement of noise-like undersampling artifacts.

Figure 2 illustrates the reconstruction performance of FA, MD, and angular deviation of the primary eigenvector ($D_\alpha$), of which the RMSEs were found to increase with the acceleration factor. A better reconstruction fidelity is generally obtained in data with higher SNR at a given acceleration factor.

Comparisons of the reconstruction performance between that of DCS and the basic CS for simulated data with SNR of 40 dB are illustrated in Figure 3. For a given acceleration factor, the sampling masks were kept identical between the two methods, of which the parameters in the algorithm were optimized based on numerous trials to achieve the smallest RMSE. Results show that DCS generally yields better reconstructions with lower RMSE compared with those of basic CS for all acceleration factors, thus demonstrating the superiority of the DCS-based approach.
Maps of FA, MD, color-coded FA, and helix angle of the rat heart sample at acceleration factors of 2, 3, 4, 5, and 6 are shown in Figure 4. The images derived from the fully sampled data are included as the references. The reconstructed maps of DTI indices are found to be visually comparable with the references at acceleration factors up to 4, with major information qualitatively preserved and negligible artifacts. At acceleration factor of 5, the image quality degrades to some extent with the presence of reconstruction artifacts observed on maps of FA, MD, and color-coded FA, and spatial blurring on helix angle map. The reconstruction performance further deteriorates with the acceleration factor increasing to 6.

Pixel-wise correlations of the reconstructed DTI indices with respective gold standards derived from the fully sampled data at varied acceleration factors are shown in Figure 5. Red dotted line with slope of 1 represents the ideal correlation. A relatively good correlation is qualitatively preserved at acceleration factors up to 4. Beyond this, the correlation is found to be apparently degraded with more scatter dots deviating from the red dotted lines.

Table 1 summaries the RMSEs of FA, MD, and $\Delta z$ respectively of the entire LV myocardium measured using the DCS-based and the repetition-reduced methods, respectively. The values obtained from 10-repetition fully sampled data were listed as gold standards. As myocardial fiber orientation was transmural location dependent and varied throughout the myocardium wall, only RMSE was included for the assessment of reconstruction accuracy of fiber orientation. However, the reconstruction accuracy progressively decreases with the acceleration factor, which is probably related to the appearance of reconstruction artifacts or spatial blurring at large acceleration factor.

At given acceleration factors of 2 and 5, DTI indices were respectively estimated by the repetition-reduced and DCS-based methods, and correlated with the gold standards obtained from the fully sampled data with 10 repetitions to evaluate the reconstruction accuracy (Fig. 6. Reconstruction performance of the DCS-based method is generally observed to be superior to that of the

FIG. 3. Comparisons of the reconstruction performance between that of DCS and basic CS on simulated data with SNR of 40 dB at different acceleration factors ($R$). The solid and dotted lines are for the DCS-based and CS-based methods, respectively. The unit of MD is $10^{-3}$ mm$^2$/s.

FIG. 4. Maps of fractional anisotropy (FA), mean diffusivity (MD), color-coded FA, and helix angle reconstructed from the fully sampled k-space data as well as undersampled DWIs with acceleration factors ($R$) of 2, 3, 4, 5, and 6. The reconstructed maps of DTI indices are found to be visually comparable with the references at acceleration factors up to 4, with major information qualitatively preserved and negligible artifacts. At higher acceleration factors beyond 4, the image quality degrades to some extent with presence of spatial blurring and reconstruction artifacts (the bright stripes in FA and color-coded FA maps, and dark ones in MD maps with representative artifacts indicated by red arrows). The unit of MD is $10^{-3}$ mm$^2$/s, and helix angle is degree ($^\circ$). For color-coded FA maps, red represents direction of left–right, green for up–down, and blue for in–out.
repetition-reduced method with more dots concentrating tightly around the ideal correlations for all the investigated DTI indices. Quantitative results show that the DCS-based method yields smaller RMSE at both acceleration factors, except in the case of $R = 5$, for which slightly larger RMSE of MD is observed.

**DISCUSSION**

DCS exploits both intra- and inter-signal correlation structures and enables new distributed coding algorithms for multisignal ensembles (42). By exploiting the redundancy across multiple signals, acceleration beyond only using sparsity of a single signal could be achieved more efficiently (44,45). DCS has been demonstrated to improve reconstruction performance or acceleration factor by fully using joint sparsity property along multiple channels in parallel imaging, cardiac perfusion MRI, and noncontrast enhanced MRA (44–46,52). In DTI, directional diffusion weightings only modulated the magnitude of DWIs, among which joint sparsity exists in the sparse transform domain. It would be expected that a larger acceleration and better performance could be achieved by simultaneously reconstructing DWIs compared with the basic disjoint CS method (i.e., reconstructing each DW image individually without using their joint sparsity property). Our simulation results have quantitatively confirmed this belief.

In this study, reconstruction performance was demonstrated to be determined by both the SNR and the acceleration factor. Generally, data with higher SNR was found to yield better estimation of DTI indices than those with more noise contamination. For a given dataset, the RMSE has been found to increase steadily with the acceleration factor. This may be due to the appearance of reconstruction artifacts at large undersampling rates.

These findings were also clearly observed in the reconstructed DTI index maps measured from the experimental heart sample data. The SNR on averaged DWIs of the 10-repetition fully sampled data was $\approx 45$ dB. Both qualitative and quantitative results exhibited relatively precise estimation of FA, MD, and fiber orientation until an acceleration factor of 4. Whereas, at larger accelerating factors, reconstruction artifacts on maps of FA, MD, and color-coded FA tended to be pronounced, leading to gradual elevation of respective RMSEs. Concurrently, spatial blurring on helix angle maps became more noticeable with increase of the acceleration factors as observed.

![Image](https://example.com/image.png)

**FIG. 5.** Pixel-wise correlations of the reconstructed DTI indices (y-axis), including fractional anisotropy (FA), mean diffusivity (MD), and helix angle, with respective gold standards derived from the fully sampled data (x-axis) at varied acceleration factors ($R$). Red dotted line with slope of 1 represents the ideal correlation. A relatively good correlation is qualitatively preserved at acceleration factors up to 4. Beyond this, the correlation is found to be apparently degraded with more scatter dots deviating from the red dotted lines. Among the investigated DTI indices, correlation of helix angle is the least sensitive to the influence of acceleration factor and usually exhibits good performance. The scale of MD is $10^{-3} \text{ mm}^2/\text{s}$, and helix angle is degree ($^\circ$). (Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.)

**Table 1**

<table>
<thead>
<tr>
<th>Repetition number</th>
<th>FA</th>
<th>MD ($10^{-3} \text{ mm}^2/\text{s}$)</th>
<th>$\Delta\varphi$ ($^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R = 2$</td>
<td>0.035</td>
<td>0.019</td>
<td>4.854</td>
</tr>
<tr>
<td>$R = 3$</td>
<td>0.043</td>
<td>0.027</td>
<td>5.719</td>
</tr>
<tr>
<td>$R = 4$</td>
<td>0.052</td>
<td>0.034</td>
<td>6.422</td>
</tr>
<tr>
<td>$R = 5$</td>
<td>0.061</td>
<td>0.044</td>
<td>6.919</td>
</tr>
<tr>
<td>$R = 6$</td>
<td>0.068</td>
<td>0.050</td>
<td>7.272</td>
</tr>
<tr>
<td>Repetition number = 5</td>
<td>0.048</td>
<td>0.031</td>
<td>5.754</td>
</tr>
<tr>
<td>Repetition number = 2</td>
<td>0.064</td>
<td>0.033</td>
<td>8.344</td>
</tr>
</tbody>
</table>

Values measured from fully sampled data with 10 scan repetitions were regarded as gold standards.
in previous studies (53), resulting in deviation of reconstructed helix angle from a fully sampled reference. It is well known that different regions of k-space contribute unequally to image reconstruction, with low frequency components hosting coarse-scale information and high frequency representing fine structures. Based on the sampling scheme used in this study, high frequency components were significantly less acquired at high acceleration factors, consequently leading to spatial blurring due to loss of image details.

Effects of Downsampling Scheme
In this study, the individual sampling mask following the same PDF was generated for each DWI corresponding to a given acceleration factor $R$. This scheme was assumed to benefit the reconstruction accuracy by strengthening the incoherence, especially for data with large number of diffusion directions. This assumption was tested on the similar simulated phantom but with 60 diffusion directions and SNR of 40 dB. When the exactly same sampling mask was applied on each DWIs, the RMSEs of FA, MD, and $\Delta \varphi$ are 0.037, 0.062 × 10^{-3} mm^2/s, and 3.048°, respectively, at acceleration factor of 3. However, the corresponding RMSEs decreased to 0.028, 0.037 × 10^{-3} mm^2/s, and 2.792°, respectively, when using different masks for DWIs. The apparent improvement of reconstruction performance demonstrated the efficacy of the sampling strategy we have adopted. Furthermore, we also found that fully acquisition of $b_0$ image had negligible impact on total scan time, but could help to improve the reconstruction accuracy of DTI data, especially at large $b$-values where signal intensities of $b_0$ and DWIs exhibited apparent discrepancy. For example, in the experimental heart DTI data, RMSEs of FA, MD, and $\Delta \varphi$ measured from undersampled $b_0$ and DWIs at acceleration factor of 3 were 0.062, 0.049 × 10^{-3} mm^2/s, and 5.856°, respectively, which were inferior to that achieved from data with $b_0$ fully sampled (RMSEs of FA, MD, and $\Delta \varphi$ were 0.044, 0.027 × 10^{-3} mm^2/s, and 5.719°, respectively). Therefore, the sampling strategy in this study was to fully sample the k-space of $b_0$ image and reconstruct the undersampled DWIs by using the joint sparsity characteristic along diffusion directions.

Influences of Diffusion Sensitivity $b$-Value
Reconstruction performance was also tested on simulated DTI data at different diffusion strengths, such as $b$-values of 500 and 1500 s/mm^2, which altered SNRs on their DWIs. The effects of the SNR and the acceleration factor were found to be similar with those of simulated data at the $b$-value of 1000 s/mm^2, thus confirming the generalized important effect of noise and acceleration factor on image reconstruction accuracy.

Convergence Behavior
In our work, the iteration terminates when the normalized difference between the consecutive reconstructed images

![FIG. 6. Comparisons of pixel-wise correlations of DTI index estimates ($y$-axis), including fractional anisotropy (FA), mean diffusivity (MD), and helix angle, by repetition-reduced and DCS-based methods at acceleration factors ($R$) of 2 and 5, respectively, with the gold standards obtained from the fully sampled data with 10 repetitions ($x$-axis). Red dotted line with slope of 1 represents the ideal correlation. Correlation performance of the DCS-based method is generally observed to be superior to that of the repetition-reduced method with more dots concentrating tightly around the ideal correlations. The scale of MD is ×10^{-3} mm^2/s, and helix angle is degree (°). (Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.)](image)
(defined as the difference between the consecutive reconstructed images divided by the formal one of the consecutive images) is less than $\frac{1}{10^2}$. It is observed that the proposed method typically requires three outer nonlinear conjugate gradient (NLCG) iterations to achieve good performance for all DTI indices. An example of the experimental heart DTI data with $R = 4$ is shown in Figure 7. Note that the values of iteration 0 correspond to the reconstructed DWIs obtained by zero-filling the undersampled k-space. Results clearly show that the cost function and reconstruction errors decay rapidly after the first iteration of NLCG and have a negligible change after the third iteration.

Comparisons Between DCS-Based and Repetition-Reduced Methods

Using the prior of sharing the common support, the DCS method may recover images by solving a set of underdetermined equations, from which the joint sparsity constraint could help to obtain the best one among an infinite number of solutions. Whereas, the approach of simply reducing scan repetition number acquires all k-space data and applies joint regression penalties on the spatial domain to enhance the reconstruction SNR with a Huber function. The approach may better manifest its denoising capability when dealing with rather noisy data, for example, only few repetitions are allowed to acquire full k-space in the given time. In this study, when moderate noisy is involved by allowing more repetitions, it is observed that the method of acquiring undersampled data achieves better DTI reconstruction than that of acquiring fully sampled data but with fewer repetitions at equivalent acquisition time. This result agrees with the previous observation that the performance metrics of reconstruction from undersampled data was better than that of the fully encoded case within the same scan time (53,54). Although the reconstruction time of our method using orthogonal wavelet transform ($144.5s$) is longer than the repetition-reduced method ($4.2s$) due to iteratively solving Eq. [6], reconstruction accuracy of DTI indices may be more important, especially in clinical diagnosis and evaluation. With code optimization and graphic processing unit (GPU) implementation, the reconstruction speed is expected to be greatly enhanced. In addition, the optimal
fast DTI scheme may depend on specific conditions (such as SNR and acceleration rate). Optimal acceleration strategy (e.g., k-space undersampling, reduction of diffusion directions or scan repetition) will be further exploited in future study.

Limitations of This Study

In this study, the undersampled k-space data was obtained in a retrospective manner. The implementation of a feasible prospective undersampling scheme combined with tailored pulse sequence may be further developed for practical applications of DCS theory. In addition, parallel imaging technique might be included in future work. It could be intuitively expected that higher acceleration factors would be achieved by incorporating additional sensitivity or sparsity information of multiple coils (55,56).

CONCLUSIONS

In this study, the feasibility and efficacy of DCS to accelerate DTI data acquisition were investigated using both simulated and experimental data. The superiority of DCS over basic CS was confirmed with a simulation study, and the reconstruction accuracy was found to be influenced by SNR and acceleration factors. Similar observations were obtained in experimental heart sample study. The reconstructed DTI indices, such as FA, MD, and myocardial fiber orientation, were found to be qualitatively and quantitatively comparable with respective gold standards at acceleration factors up to 4. Spatial blurring and reconstruction artifacts tended to be prominent with the increasing acceleration factor, leading to gradual degradation of reconstruction performance. Furthermore, estimation accuracy of reconstructed DTI indices based on DCS technique was demonstrated to be superior to that of repetition-reduced method within equivalent scan time. In summary, this study demonstrated the feasibility and efficacy of DCS to accelerate DTI with using joint sparsity property along diffusion directions.

ACKNOWLEDGMENTS

The authors thank the reviewers for their valuable comments. They also thank Dr. Peter Z. Wu for helpful discussion.

REFERENCES